Mechanics 1

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CONTENTS

* calculations
* new syllabus

① Kinematics
- motion of particle in straight line
  - with constant a
  - with variable a

② Statics
- Resultant Force of System, in disequilibrium
- equilibrium system → limiting friction/equilibrium

def. of coeff. of friction

$F = \mu R$ or $F \leq \mu R$

③ Dynamics
- $F = ma$ → $W = mg$ → motion of particle → motion of chain of particles

④ Work Done, Energy and Power

$W = Fd \cos\theta$

$P = \frac{W}{t} = Fv$

⑤ Momentum and Impulse

* particle physics (treat as 0-d point) (translational)
  - linear $\vec{s}$
  - linear v $\left(\frac{d\vec{s}}{dt}\right)$
  - linear a $\left(\frac{d^2\vec{s}}{dt^2}\right)$

* Calculator Used

at least 570

old middle latest

Chapter 1: Kinematics

(motion on a straight line)

- DISPLACEMENT / DISTANCE
- VELOCITY
- ACCELERATION

- EQUATIONS OF MOTION
(assuming constant acceleration)

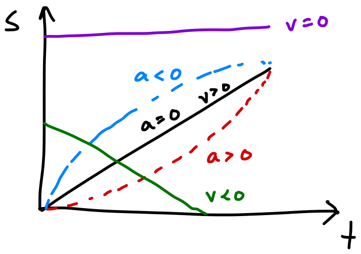
① $s = \frac{1}{2}(u+v)t$

② $v = u + at$

③ $v^2 = u^2 + 2as$

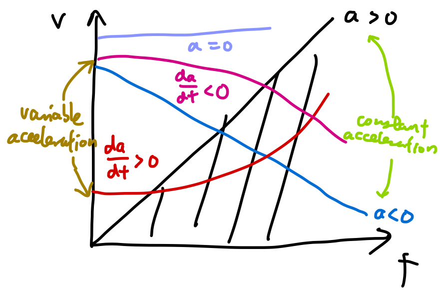
④ $s = ut + \frac{1}{2}at^2$

- s-t GRAPH



$\frac{ds}{dt} = v$

- v-t GRAPH



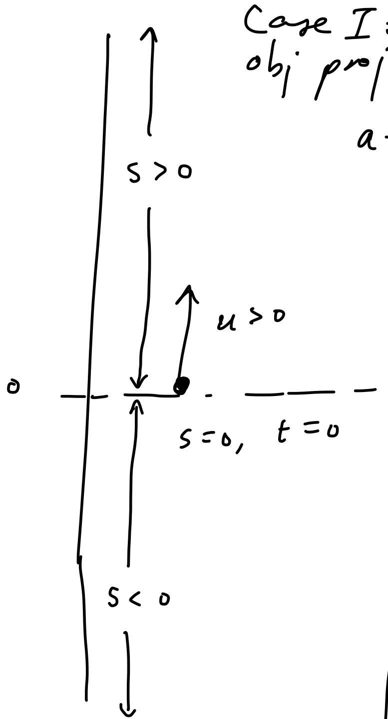
$\frac{dv}{dt} = a$

Area under graph $= s$.

-SOLVING PROBS w/ CONSTANT a

① Use the constant acceleration formulae

② Use the v-t graphs.



Case I:
obj projected upwards
$a = -10$ (constant)

$v = u + at$

$\Rightarrow v = u - gt$

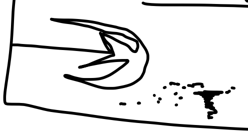
$$s = ut - \frac{1}{2}gt^2$$

$$2s = 2ut - gt^2 \Rightarrow \text{quadratic in } t$$

$$gt^2 - 2ut + 2s = 0$$

$$t = \frac{2u \pm \sqrt{4u^2 - 4(g)(2s)}}{2g}$$

$$\boxed{t = \frac{2u \pm \sqrt{4u^2 - 8gs}}{2g}}$$



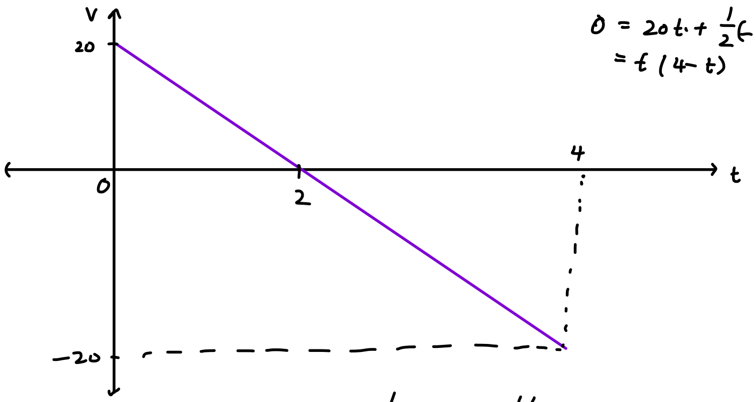
If $s>0$, t can take 2 +ve values.

If $s=0$, $t = 0$ or $t = \frac{2u}{g}$.

If $s<0$, t takes 1 +ve value (accept)
& 1 -ve value (reject)

Velocity time graph (case I)

Ex. If obj is projected w/ a speed of $20\ ms^{-1}$ vertically upwards from the ground.



$s = ut + \frac{1}{2}at^2$

$0 = 20t + \frac{1}{2}(-10)t^2$

$= t(4-t)$

$v = u + at$

$v = 20 + (-10)(4)$

$s = ut + \frac{1}{2}at^2$

$0 =$

Motion of a p w/ variable a

→ CANNOT use constant acceleration formulae!

3 cases

a) $s = f(t)$

→ $v = \frac{ds}{dt}$

→ $a = \frac{dv}{dt}$ or $\frac{d^2s}{dt^2}$

b) $v = f(t)$

→ $s = \int v\, dt$

→ $a = \frac{dv}{dt}$

c) $a = f(t)$

→ $v = \int a\, dt$

→ $s = \int v\, dt$

*remember when doing "$\int$", add "+C"!

Chapter 2: Statics

"static" — stationary / at rest

resultant forces

1) resultant of two F
   → Δ or ▱ law of addition

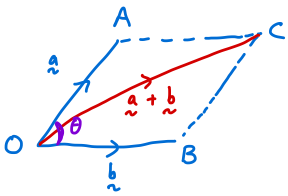
2) resultant of more than 2F
   → res. of F

A Force has a magnitude & direction
→ hence F is a vector → represented by a directed line segment eg $\overrightarrow{AB}$

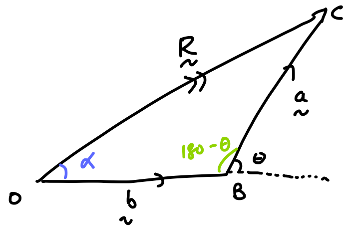
→ represented in algebraic form. $\underset{\sim}{F} = (3\underset{\sim}{i} + 4\underset{\sim}{j})$ N

*to find the resultant of 2 F acting on a p. where the forces are represented by a directed line segment.

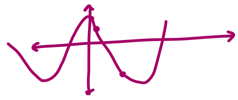
① ▱ law of ⊕
→ if 2 F, acting on a p at O, be represented in magnitude & dir. by 2 directed line segments $\overrightarrow{OA}$, $\overrightarrow{OB}$ drawn from O.



→ to find magn. & dir. of the resultant.



-let R = magn. of resultant force.



$\therefore |\underset{\sim}{R}| = \sqrt{|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - 2|\underset{\sim}{a}||\underset{\sim}{b}|\cos(180-\theta)}$

$|\underset{\sim}{R}| = \sqrt{|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + 2|\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta}$

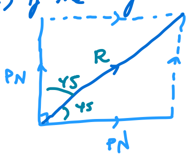
- let α be direction of the resultant force.

sine rule : $\dfrac{|\underset{\sim}{a}|}{\sin\alpha} = \dfrac{|\underset{\sim}{R}|}{\sin(180-\theta)}$

SPECIAL CASES

① If 2 forces ⊥ to each other.

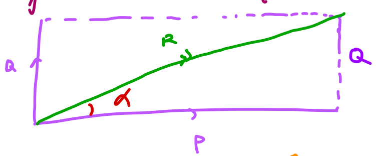
a) if the 2 forces are equal in magnitude. (square)



$R = \sqrt{2}P$

The resultant F makes an ∡ of 45° with one of the PN forces in the system

b) if the 2 F are not equal (rect)



$R = \sqrt{P^2 + Q^2}$

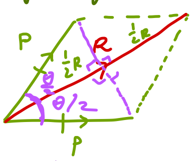
$\alpha = \tan^{-1}\left(\frac{Q}{P}\right)$

*must be in words!

The resultant F makes an angle α with the force P

② If forces X ⊥ to each other.

a) if 2 forces are equal (rhombus)



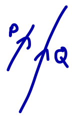
$\cos\frac{\theta}{2} = \dfrac{\left(\frac{R}{2}\right)}{P}$

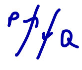
$R = 2P\cos\frac{\theta}{2}$

→ Resultant force makes an angle of $\frac{1}{2}\theta$ w/ the direction of one of the PN forces.

b) if 2 forces are not equal (parallelogram)
- general parallelogram
- solve using cosine & sine rules

③ F are //.

a)  $R = \underset{\sim}{P} + \underset{\sim}{Q}$

b)  $R = \underset{\sim}{0}$

c)  $R = \underset{\sim}{P} - \underset{\sim}{Q}$

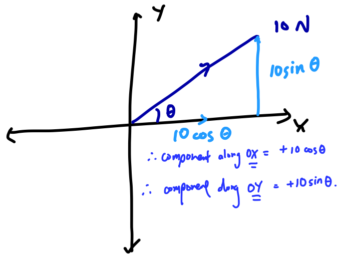
Resolution of a force.

→ every F can be resolved into 2 ⊥ components by trig.

eg²

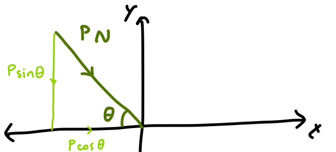
eg³

eg¹



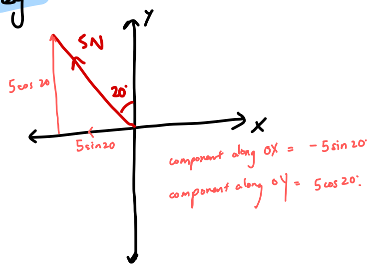
∴ component along $\underline{OX}$ = $+10\cos\theta$

∴ component along $\underline{OY}$ = $+10\sin\theta$.



component along OX = $P\cos\theta$

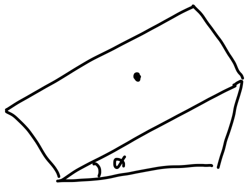
component along OY = $-P\sin\theta$



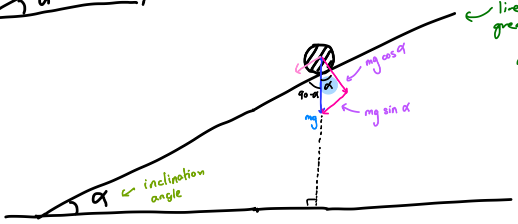
component along OX = $-5\sin 20°$

component along OY = $5\cos 20°$.

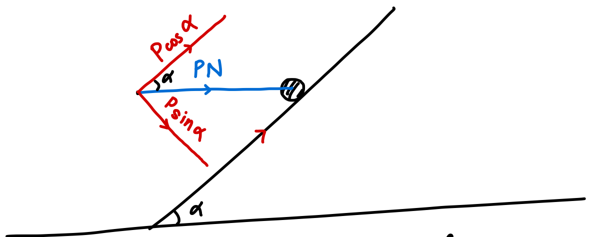
inclined plane



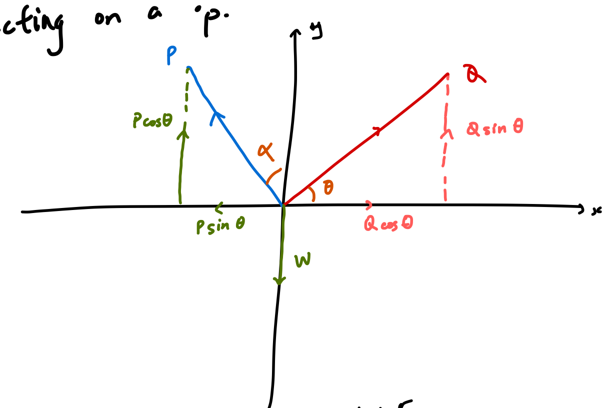
→ to resolve the weight // & ⊥ to a line of the greatest slope of the plane.



the line of the greatest slope/ inclined plane. → we use this at horizontal.



To find the resultant of a system of coplanar F acting on a 'p.



Q: Find the magn & dir of the resultant F.

method:

① Resolve all the F into 2 ⊥ components.

②

	(⟷)	(↑)
Q	$+Q\cos\theta$	$+Q\sin\theta$
P	$-P\sin\alpha$	$+P\cos\alpha$
W	0	$-W$
Σ	X	Y

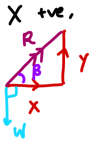
$X = Q\cos\theta - P\sin\alpha$

$Y = Q\sin\theta + P\cos\alpha - W$

↓

4 cases

① X +ve, Y +ve

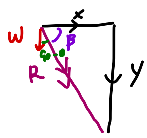


· 1st quadrant

$\beta = \tan^{-1}\left(\frac{Y}{X}\right)$

→ dir. of R makes an ∡ of $(90+\beta)$ anticlockwise w/ the dir. of the W.N force in the system.

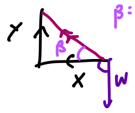
② $X<0,\ Y<0$



$\beta = \tan^{-1}\left(\frac{Y}{X}\right)$

∴ dir of R makes an ∡ of $(90-\beta)$ anticlockwise w/ the dir. of the W.N force in the system.

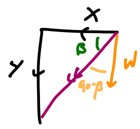
③ $X<0,\ Y>0$



$\beta = \tan^{-1}\left(\frac{Y}{X}\right)$

dir. of R makes an ∡ of $90+\beta$ clockwise w/ the dir of W.

④ $X<0,\ Y<0$



$\beta = \tan^{-1}\left(\frac{Y}{X}\right)$

∴ dir of R makes an angle of $90-\beta$ clockwise w/ the dir of W.

Statics of a particle.

equilibrium conditions :

→ the resultant $F = 0$.

recall : resolve → = the sum of components in the horizontal

resolve ↑ = the sum of components in the vertical, or ⊥ to (→).

To find the resultant of the whole system : $R = \sqrt{X^2+Y^2}$.

* if system is in eq, $F_R = 0$.

i.e. $R = 0 \Rightarrow \sqrt{X^2+Y^2} = 0$

$\therefore X^2+Y^2 = 0$.

$X^2 \geq 0,\ Y^2 \geq 0$ for $X, Y \in \mathbb{R}$

$\therefore X^2 = 0,\ Y^2 = 0$.

$\therefore X = 0,\ Y = 0$.

* ⇒ we can equate the components of the forces in 2 ⊥ directions.

types of forces

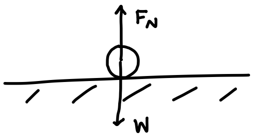
① Weight

$W = mg$, $g = 10$

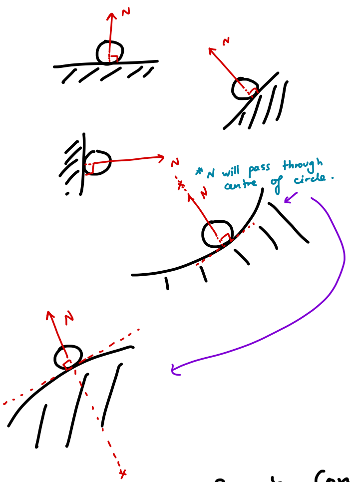
② Contact Forces (normal reaction & friction)

R or N ↑   F ↑

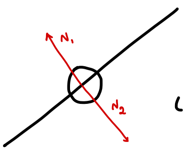
⇒ Newton's 3rd Law: every action has an equal and opp. reaction.



i) normal reaction.



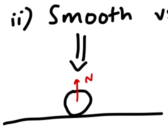
* N will pass through centre of circle.



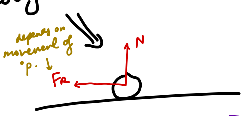
(thredded onto a wire).

* could be either N_1 or N_2. it depends on the system.

ii) Smooth vs Rough Contact.

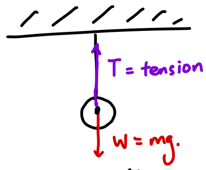


- no frictional force.
- ∴ only contact F is NCF.



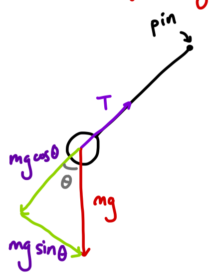
- contact $F = N + F_R$.
- $|R| = \sqrt{N^2 + F_R^2}$

③ Forces of Attachment.

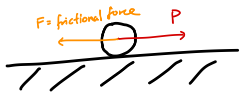


- taut : there is tension
- slack : there is no tension.
- break : tension exceeds breaking strength.

* all strings in MI are INEXTENSIBLE / INELASTIC.



④ Friction



⇒ friction acts to oppose relative motion.

⇒ if book is at rest, $P = F$.

if P ↑, book will be on the point of moving. → "limiting equilibrium"

eventually, P will reach a cf pt, where the book will start sliding.

⇒ F has reached its limiting value :

$F = \mu R$ μ = coeff. of friction, R = normal contact force

μ ↑, roughness ↑
μ ↓, roughness ↓.

① p in eq. $F_R \leq \mu R$.

② p in limiting eq. $F_R = \mu R$

③ particle starts sliding. $F_R = \mu R$. ← limit!

Chapter 3: Dynamics

→ straight line motion of a •p under the action of a syst. of coplanar F.

☆ Newton's 2nd Law:
the force F applied to a •p is proportional to the mass, m of the •p & the acceleration produced. $F = ma$. → translational motion

m : mass ⇒ linear inertia

a : linear acceleration ⇒ $\ddot{x}$ or $\dot{v}$

F : resultant of the forces acting on the •p, in the direction of motion.

Steps to solving problems involving dynamics.

① Draw a diagram showing the forces acting on a •p.

② Resolve forces into 2 ⊥ directions.
- one // to direction of motion
- one ⊥ to direction of motion

③ Since there is no a ⊥ to direction of motion, we can equate components of F ⊥ to the direction of motion.

$\overset{\rightarrow\rightarrow}{\bigcirc}$ •p moving (↔) ⇒ equate (↕)   vv for vertical motion.

④ Apply F = ma in the dir. of motion.

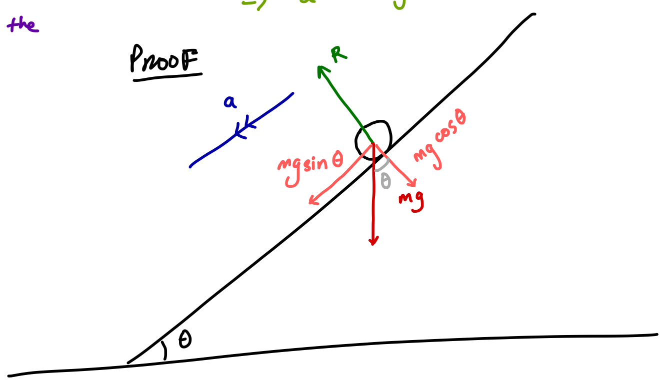
☆ When a •p is sliding down a smooth inclined plane,

⇒ $a = +g\sin\theta$ θ = ∡ of inclination of plane w/ horizontal

☆ When a •p is projected up a smooth inclined plane.

⇒ $a = -g\sin\theta$

PROOF

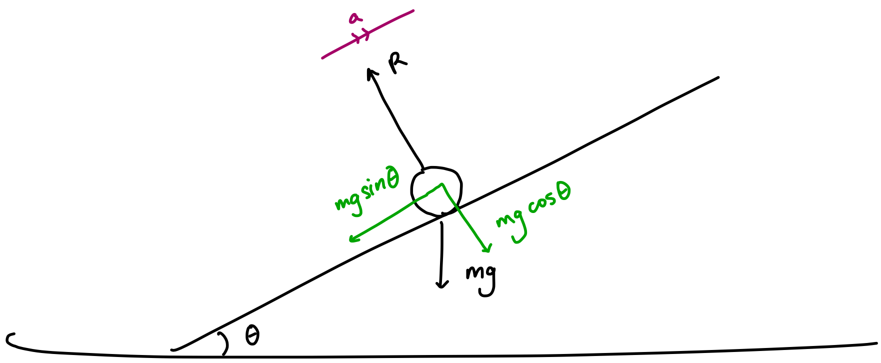


(↕) $R = mg\cos\theta$

(↗) $mg\sin\theta = ma$

$\therefore a = g\sin\theta$ ⇒ constant

⇒ can use constant acceleration formulae



(↗) $0 - mg\sin\theta = ma$

$\therefore a = -g\sin\theta$.

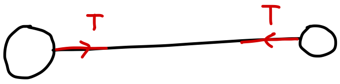
Motion of 2 connected •p

Consider 2 •p att. to the ends of a lnght inext. str:

$v_1 > v_2 \Rightarrow$ string is slack
$v_1 < v_2 \Rightarrow$ string breaks
$v_1 = v_2 \Rightarrow$ string is taut.

same speed & accel.

By N3L, forces acting on the •p will have the same magn. but opp. dir.



Problems involving pulleys

→ heavier •p will move ↓

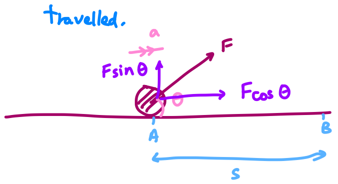
→ lighter •p will move ↑

* motions of the 2 •p are not independent bc they are connected by the string.

Chapter 4:
Work, Energy and Power

WORK DONE

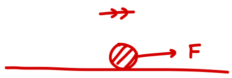
↳ defined as the component of the F in the direction of the motion times the distance travelled.



$W = sF\cos\theta$

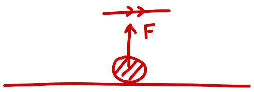
* special cases:

①



$W.D. = F\cos 0 \times s$
$= Fs$

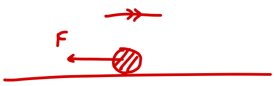
②



$W.D = F\cos 90 \times s$
$= 0$

→ forces ⊥ to the dir. of motion do not do work.

③



$W.D = F\cos 180 \times s$
$= -Fs.$

"-ve" indicates work is done against F.

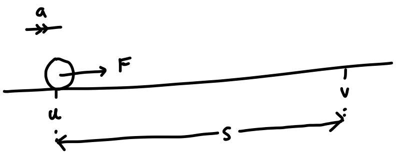
MECHANICAL ч

KE

→ •p is moving ⇒ KE

$K.E = \frac{1}{2}mv^2$

→ consider:



W.D by the FN force on the •p $= F \times s$

By N2L : $F = ma$ — ①

We know $v^2 = u^2 + 2as$

$\therefore a = \frac{v^2 - u^2}{2s}$ — ②

② → ① $F = m\left(\frac{v^2-u^2}{2s}\right)$

$\underbrace{Fs}_{\text{work done by the force}} = \underbrace{\frac{1}{2}mv^2}_{KE_{final}} - \underbrace{\frac{1}{2}mu^2}_{KE_{initial}}$

* ↑K.E = $E_{k_f} - E_{k_i}$ $\frac{1}{2}m(v^2-u^2)$

↓K.E = $E_{k_i} - E_{k_f}$ $\frac{1}{2}m(u^2-v^2)$

GPE

→ the ч a body possesses by virtue of its pos in a gravitational field.

* $G.P.E = mgh.$

*
↑GPE $= mg|\Delta h|$
↓GPE $= -mg|\Delta h|$

ENERGY PRINCIPLES

i) Work-Energy principle

↳ W.D by a F = ↑ E

↳ W.D against a F = ↓ E

ii) Principle of Conservation of Energy

↳ if there is no impulse, collision, friction, resistance; the total mechanical E of the system $\boxed{=}$.

a) $\Sigma E_i = \Sigma E_f$

$\boxed{\begin{array}{l} \text{b) } \uparrow GPE = \downarrow KE \\ \downarrow GPE = \uparrow KE \end{array}}$

c) $\Sigma E_{mechanical}$ is constant.

*motion may be non-linear.

POWER

↳ defined as the rate of work done.

$$\boxed{P = \frac{W.D}{t}}$$

*motion of a moving vehicle.

↳ if the engine of a vehicle is producing a driving force of "D", when the vehicle has speed "v", then the W.D / second is

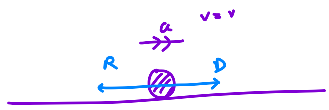
$P = \frac{W}{t}$

$= \frac{D \times s}{t}$

$\boxed{P = Dv.}$

* to find the acceleration of the vehicle at a given instant:

① On level road

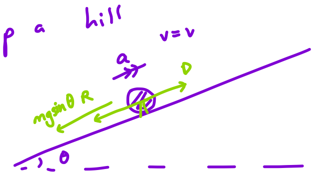


(→) $D - R = ma$

$\boxed{\frac{P}{v} - R = ma}$

*not constant acceleration.

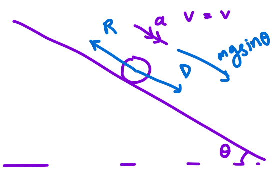
② Up a hill



↗ $D - mg\sin\theta - R = ma$

$\boxed{\frac{P}{v} - mg\sin\theta - R = ma.}$

③ Down a hill



↘ $D + mg\sin\theta - R = ma$

$\boxed{\therefore \frac{P}{v} + mg\sin\theta - R = ma}$

iii) W.D by the driving force (= W.D by the car engine)

$= \underbrace{\text{gain in energy}}_{GPE \& KE} + \underbrace{\text{W.D against R.}}_{R\times s \text{ or } P\times t}$

↳ def^n of work done: Ds → use if D is a constant

↳ work done by D: Pt → use if P is constant

Chapter 5: Momentum

MOMENTUM

defⁿ → $p = mv$ = mass × velocity

v vector ∴ p is a vector

(has magn. & dir.)

IMPULSE

defⁿ → impulse for a force F that acts on a body for t_1 to t_2

$$= \int_{t_1}^{t_2} F \, dt.$$

$$\Rightarrow I = \int_{t_1}^{t_2} F \, dt.$$

IMPULSE - MOMENTUM PRINCIPLE

→ suppose a force $\underline{F}$ acting on a body of mass m for time t, w/ initial vel. u & final vel. v.

$$\rightarrow \quad I = \int_0^t F \, dt$$

$$= F \int_0^t 1 \, dt$$

$$= F[t]_0^t$$

$$= F(t-0)$$

$$I = Ft.$$

* if F is a constant.

since F is a vector, I is also a vector.

① dir. of F = dir. of I

$$I = \int_0^t F \, dt$$

$$I = \int_0^t ma \, dt$$

$$I = m \int_0^t \frac{dv}{dt} \, dt$$

$$= m \int_u^v 1 \, dv$$

$t=0,\ v=u$
$t=t,\ v=v$

$$= m[v]_u^v$$

$$= mv - mu$$

$$\boxed{\therefore Ft = mv - mu.}$$

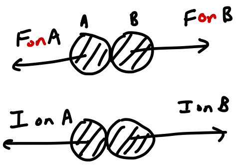
momentum after − momentum before.

★ To use this principle:

→ we must know the dir. of I; &

→ we will always take the dir. of I is +ve.

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM.



By impulse-momentum principle,

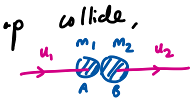
For A: $(\overset{I}{\leftarrow})$ $I = m_1(-v_1) - m_1(-u_1)$

$= -m_1v_1 + m_1u_1$ — ①

For B: $(\overset{I}{\rightarrow})$ $I = m_2v_2 - m_2u_2$ — ②

When two ·p collide,

Before —



After —

①=②

$$\Rightarrow -m_1v_1 + m_1u_1 = m_2v_2 - m_2u_2$$

$$\therefore \underbrace{m_1u_1 + m_2u_2}_{\text{total initial momentum}} = \underbrace{m_1v_1 + m_2v_2}_{\text{total final momentum}}.$$

To use PCM

→ fix one direction to be +ve.

(convention: → = +ve)

$\boxed{\text{* coalesce: two op stick together after the collision.}}$